

Simultaneous quantum state teleportation via the locked entanglement channel

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We present a simultaneous quantum state teleportation scheme, in which receivers can not recover their quantum state separately. When they want to recover their respective quantum state, they must perform an unlocking operator together.

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Quantum entanglement plays an important role in various fields of quantum information, such as quantum computation, quantum cryptography, quantum teleportation and dense coding etc. Quantum teleportation is one of the most important applications of quantum entanglement. In quantum teleportation process, an unknown state can be transmitted from a sender Alice to a receiver Bob without transmission of carrier of quantum state. Since Bennett et al [1] presented a quantum teleportation scheme, there has been great development in theoretical and experimental studies. Now quantum teleportation has been generalized to many cases [2, 3, 4, 5, 6, 7, 8, 9]. Moreover, quantum teleportation has been demonstrated with the polarization photon [10] and a single coherence mode of fields [11] in the experiments. The teleportation of a coherent state corresponding to continuous variable system was also realized in the laboratory [12].

In the original proposal, the teleportation of a single qubit $|\phi\rangle = a|0\rangle + b|1\rangle$ is executed as follows: Alice and Bob initially share an EPR pair as a quantum channel. Alice performs a joint measurement on the composed system (qubit to be teleported and one of the entangled pair). She transmits the outcome to Bob through a classical channel. Bob applies a corresponding unitary operation on his particle of the entangled pair, which is chosen in accordance with the outcome of joint measurement. The final state of Bob's qubit is completely equivalent to the original unknown state.

If Alice wants to teleport the quantum state $|\phi_1\rangle$ to Bob and teleport $|\phi_2\rangle$ to Charlie, obviously two EPR pairs are required. One is shared between Alice and Bob; the other is shared between Alice and Charlie. The quantum state teleportation can be completed by Bennett's protocol. But if Bob and Charlie want to receive their respective quantum state simultaneously, how do they complete the teleportation? In this paper, we will present a simultaneous quantum state teleportation scheme. In this scheme, the receivers Bob and Charlie can synchronously recover the quantum state which Alice

teleported to them respectively by locking their quantum channels.

To present our scheme clearly, let's first begin with simultaneous quantum state teleportation between one sender and two receivers.

The quantum states of two qubits T_1 and T_2 to be teleported are as follows

$$\begin{aligned} |\phi_1\rangle_{T_1} &= \alpha_1|0\rangle_{T_1} + \beta_1|1\rangle_{T_1}, \\ |\phi_2\rangle_{T_2} &= \alpha_2|0\rangle_{T_2} + \beta_2|1\rangle_{T_2}, \end{aligned} \quad (1)$$

Alice wants to teleport $|\phi_1\rangle$ to Bob and $|\phi_2\rangle$ to Charlie simultaneously. Suppose that Alice, Bob and Charlie share two EPR pairs denoted as

$$\begin{aligned} |EPR\rangle_1 &= \frac{1}{\sqrt{2}}(|00\rangle_{A_1B} + |11\rangle_{A_1B}), \\ |EPR\rangle_2 &= \frac{1}{\sqrt{2}}(|00\rangle_{A_2C} + |11\rangle_{A_2C}), \end{aligned} \quad (2)$$

where A_1 and A_2 belong to Alice, B and C belong to Bob and Charlie respectively. Then the quantum state of the joint system (qubits to be teleported and two EPR pairs) can be written as

$$\begin{aligned} |\Phi\rangle &= (\alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \alpha_2\beta_1|10\rangle + \beta_1\beta_2|11\rangle)_{T_1T_2} \\ &\otimes \frac{1}{2}(|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle)_{A_1A_2BC}. \end{aligned} \quad (3)$$

The scheme of simultaneous teleportation consists of the following five steps.

(S1) Locking the quantum channels

In this step of teleportation, Bob and Charlie perform a joint unitary transformation

$$U_{BC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \quad (4)$$

on the particles B and C . After that, the state of the total system becomes

$$\begin{aligned} |\Phi'\rangle &= (\alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \alpha_2\beta_1|10\rangle + \beta_1\beta_2|11\rangle)_{T_1T_2} \\ &\otimes \frac{1}{2\sqrt{2}}(|0000\rangle + |0011\rangle + |0101\rangle + |0110\rangle \\ &\quad + |1000\rangle - |1011\rangle + |1101\rangle - |1110\rangle)_{A_1A_2BC}. \end{aligned} \quad (5)$$

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(S2) Performing Bell-basis measurement

Alice performs a projective measurement on A_1T_1 and one on A_2T_2 in the Bell-basis $\{|\Psi_m\rangle, m = 0, 1, 2, 3\}$, where

$$\begin{aligned} |\Psi_0\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \quad |\Psi_1\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \\ |\Psi_2\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle), \quad |\Psi_3\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle). \end{aligned} \quad (6)$$

Let σ_j be one member of the set of rotation matrices

$$\{I, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\} \quad (7)$$

composed of the identity operator and three Pauli spin operators. It is easy to prove that Eq.(5) can be rewritten as

$$|\Phi'\rangle = \sum_{m,n} |\Psi_m\rangle_{A_1T_1} |\Psi_n\rangle_{A_2T_2} U_{BC} \sigma_m |\phi_1\rangle_B \otimes \sigma_n |\phi_2\rangle_C. \quad (8)$$

After the projective measurement, the state of particles B and C collapses into

$$|\Psi\rangle_{BC} = U_{BC} \sigma_m |\phi_1\rangle_B \otimes \sigma_n |\phi_2\rangle_C, \quad (9)$$

which corresponds to the measurement result $|\Psi_m\rangle, |\Psi_n\rangle$.

(S3) Transmitting the measurement outcome

After performing the Bell-basis measurement, Alice transmits the outcome of measurement (i.e. m and n) to Bob and Charlie.

(S4) Unlocking the quantum state

According to Eq.(9) the quantum state of particle B and C is locked, so Bob and Charlie must "unlock" B and C firstly. In order to recover the quantum states which are teleported from Alice, Bob and Charlie perform a unitary operator U_{BC}^+ on the particles B and C , where

$$U_{BC}^+ = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}. \quad (10)$$

After that the state of particle B and C is transformed into

$$\begin{aligned} &|\Psi'\rangle_{BC} \\ &= U_{BC}^+ U_{BC} \sigma_m |\phi_1\rangle_B \otimes \sigma_n |\phi_2\rangle_C = \sigma_m |\phi_1\rangle_B \otimes \sigma_n |\phi_2\rangle_C. \end{aligned} \quad (11)$$

(S5) Recovering the quantum state

Bob and Charlie perform a local unitary operator σ_m and σ_n on particles B and C respectively, then they will obtain $|\phi_1\rangle$ and $|\phi_2\rangle$ immediately.

In the following, we will discuss why we call the step (S1) "locking" the quantum channel. It is easy to verify that U_{BC} can be realized by a Hadamard operator on particle B and a CNOT operator (B is control qubit and C is target qubit). Apparently U_{BC} is a non-local operator. Now, is there any entanglement between B and C after Bob and Charlie make the unitary operator U_{BC} on them? The answer is negative. To investigate the entanglement between B and C , we employ

the Peres-Horodecki criterion [13, 14] for two qubits that their density operator ρ is separable if and only if its partial transposition is positive. The partial transposition of ρ is defined as

$$\rho^T = \sum_{ijkl} \rho_{jikl} |i\rangle\langle j| \otimes |k\rangle\langle l|, \quad (12)$$

where $\rho = \sum_{ijkl} \rho_{ijkl} |i\rangle\langle j| \otimes |k\rangle\langle l|$. After transformation, the reduced density operator of (B, C) is given as

$$\rho'_{BC} = \frac{1}{4}I. \quad (13)$$

The partial transposition of ρ'_{BC} has only positive eigenvalues $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$. The result implies that there is no entanglement between B and C . In fact, before transformation the reduced density operator of (B, C) is $\rho_{BC} = \frac{1}{4}I$. For any unitary operator U we always have

$$\rho'_{BC} = U^+ \rho_{BC} U = \rho_{BC} = \frac{1}{4}I, \quad (14)$$

i.e. there is no entanglement between B and C for arbitrary unitary operator U . Notice that after transformation, the quantum channels become

$$\begin{aligned} |\Psi'\rangle &= \frac{1}{2\sqrt{2}} (|0000\rangle + |0011\rangle + |0101\rangle + |0110\rangle \\ &\quad + |1000\rangle - |1011\rangle + |1101\rangle - |1110\rangle)_{A_1A_2BC} \\ &= \frac{1}{2\sqrt{2}} [(|00\rangle + |10\rangle)_{A_1B} \otimes (|00\rangle + |11\rangle_{A_2C}) \\ &\quad + (|01\rangle - |11\rangle)_{A_1B} \otimes (|01\rangle + |10\rangle_{A_2C})]. \end{aligned} \quad (15)$$

Eq. (15) shows that $|\Psi'\rangle$ is maximally entangled state of A_1B and A_2C . In other words, the function of U_{BC} is not to entangle (B, C) but (A_1B, A_2C) . In some sense, U_{BC} is like a "lock" which prevents Bob and Charlie from recovering their quantum states separately. More surprisingly, the initial state of (A_1, B) is maximally entangled, while after performing the unitary transformation U_{BC} the reduced density operator ρ'_{A_1B} reads

$$\rho'_{A_1B} = \frac{1}{4} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}. \quad (16)$$

The partial transposition of ρ'_{A_1B} has only nonnegative eigenvalues $0, 0, \frac{1}{2}, \frac{1}{2}$. The result implies that the entanglement of (A_1, B) has vanished. Similarly, the entanglement of (A_2, C) has also vanished. That is, A_1B and A_2C have no function of the quantum channels, after U_{BC} is applied on B and C particles. Alice can teleport $|\phi_1\rangle$ and $|\phi_2\rangle$ to Bob and Charlie respectively due to the entanglement between A_1B and A_2C .

It is worthy pointing out that "locking" the quantum channels can be completed by Alice. On the one hand, before distributing the entanglement pairs, Alice does the transformation U_{BC} on the two qubits which will be sent to Bob and Charlie, i.e. on qubits B and C . The advantage of which is that Bob and Charlie need not come together to lock the states. They only need to come

together at the time they want to unlock it. On the other hand, Alice may also make unitary operator $U_{A_1 A_2}$ on qubits A_1 and A_2 after distributing the entanglement pairs

$$\begin{aligned} & U_{A_1 A_2} \otimes I_{BC} \frac{1}{2}(|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle)_{A_1 A_2 BC} \\ &= \frac{1}{2\sqrt{2}}(|0000\rangle + |0011\rangle + |0101\rangle + |0110\rangle + |1000\rangle \\ &\quad - |1011\rangle + |1101\rangle - |1110\rangle)_{A_1 A_2 BC}, \end{aligned} \quad (17)$$

where

$$U_{A_1 A_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}. \quad (18)$$

It is straightforward to generalize the above protocol to n receivers who recover the quantum states simultaneously. Suppose Alice wants to teleport $|\phi_i\rangle$ to Bob i ($i = 1, 2, \dots, n$), Alice and every receiver share an EPR pair denoted as

$$|EPR\rangle_i = \frac{1}{\sqrt{2}}(|00\rangle_{A_i B_i} + |11\rangle_{A_i B_i}), i = 1, 2, \dots, n. \quad (19)$$

The steps in this case are similar to that of two receivers.

It is only necessary to replace locking operator $U_{B_1 B_2}$ and unlocking operator $U_{B_1 B_2}^+$ by unitary operator

$$U_{B_1 B_2 \dots B_n} = \Pi_{i=2}^n (\text{CNOT})_{B_1 B_i} H_{B_1} \quad (20)$$

and $U_{B_1 B_2 \dots B_n}^+$ respectively. Here H_{B_1} is a Hadamard operator on the qubit B_1 .

Without difficulty one can figure out the network for locking the n quantum channels, for saving space we do not depict it here.

In summary, we present a simultaneous quantum state teleportation scheme, in which receivers can not recover their quantum state separately. When they want to recover their quantum states, they must perform a unlocking operator together.

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